

# 3-Neutrino Mass Spectrum from Combining Seesaw and Radiative Neutrino Mass Mechanisms

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## Abstract

We extend the Standard Model by adding a second Higgs doublet and a right-handed neutrino singlet with a heavy Majorana mass term. In this model, there are one heavy and three light Majorana neutrinos with a mass hierarchy  $m_3 \gg m_2 \gg m_1$  such that only  $m_3$  is non-zero at the tree level and light because of the seesaw mechanism,  $m_2$  is generated at the one-loop and  $m_1$  at the two-loop level. We show that the atmospheric neutrino oscillations and large mixing MSW solar neutrino transitions with  $\Delta m_{\text{atm}}^2 \simeq m_3^2$  and  $\Delta m_{\text{solar}}^2 \simeq m_2^2$ , respectively, are naturally accommodated in this model without employing any symmetry.

## I. INTRODUCTION

At present, neutrino oscillations [1,2] play a central role in neutrino physics. Recent measurements of the atmospheric neutrino flux show convincing evidence for neutrino oscillations [3] with a mass-squared difference  $\Delta m_{\text{atm}}^2 \sim 10^{-3} \div 10^{-2} \text{ eV}^2$ . It is also likely that the solar neutrino deficit finds an explanation in terms of neutrino oscillations [4], either by the MSW effect [5] with  $\Delta m_{\text{solar}}^2 \sim 10^{-5} \text{ eV}^2$  or by vacuum oscillations with  $\Delta m_{\text{solar}}^2 \sim 10^{-10} \text{ eV}^2$ . For recent reviews about neutrino oscillations see, e.g., Ref. [6].

Confining ourselves to 3-neutrino oscillations and thus ignoring the LSND result [7], neutrino flavour mixing [8] is described by a  $3 \times 3$  unitary mixing matrix  $U$  defined via

$$\nu_{aL} = \sum_{j=1}^3 U_{aj} \nu_{jL} \quad \text{with} \quad a = e, \mu, \tau, \quad (1.1)$$

where  $\nu_{aL}$  and  $\nu_{jL}$  are the left-handed components of the neutrino flavour and mass eigenfields, respectively. Then, the solar and atmospheric neutrino mixing angles are given by

$$\sin^2 2\theta_{\text{solar}} = \frac{4 |U_{e1}|^2 |U_{e2}|^2}{(|U_{e1}|^2 + |U_{e2}|^2)^2}, \quad (1.2)$$

$$\sin^2 2\theta_{\text{atm}} = 4 |U_{\mu 3}|^2 (1 - |U_{\mu 3}|^2), \quad (1.3)$$

respectively. For the small-mixing MSW solution of the solar neutrino problem,  $\sin^2 2\theta_{\text{solar}}$  is of order  $5 \times 10^{-3}$ , whereas for the vacuum oscillation and large-mixing MSW solutions this quantity is of order one [4]. Future experimental data will hopefully allow to discriminate between the different possible solutions. On the other hand, for the atmospheric neutrino oscillations the results of the Super-Kamiokande experiment give best fit values  $\sin^2 2\theta_{\text{atm}} = 0.99 \div 1$  and  $\sin^2 2\theta_{\text{atm}} \gtrsim 0.84$  at 90% CL [9].

The above-mentioned values of the oscillation parameters pose considerable problems for model builders in addition to the problem of explaining the smallness of neutrino masses. From now on we concentrate on Majorana neutrinos. There is a vast literature on models of 3-neutrino masses and mixing (see, e.g., the reviews [10–12] and also Ref. [13] and citations therein). One possibility to explain the smallness of the neutrino masses is the see-saw mechanism [14,10,15]. The other two mechanisms are obtained by extensions of the Standard Model (SM) in the Higgs sector [16] without adding any leptonic multiplets: The first one needs an extension by a Higgs triplet [17] and leads to neutrino masses at the tree level. The smallness of the neutrino masses is explained by the small triplet vacuum expectation value (VEV) which is achieved by a large mass scale in the Higgs potential (type II seesaw) [18]. The other possibility is given by purely radiative neutrino masses with the generic examples of the Zee model [19] (one-loop masses) and the Babu model [20] (two-loop masses). Examples of these types can be found, e.g., in Refs. [21,22].

In this paper we will discuss a model which combines the standard see-saw mechanism with radiative neutrino mass generation. In this framework, no other Higgs multiplets apart from scalar doublets are needed. The most general version of such a scenario with  $n_L$  lepton doublets and charged lepton singlets,  $n_R$  right-handed neutrino singlets and  $n_H$  Higgs doublets has been discussed in Ref. [23]. Here we confine ourselves to the most economic case describing a viable 3-neutrino mass spectrum, namely  $n_R = 1$  and  $n_H = 2$ . As was shown in Ref. [23] (see also Ref. [24]), this case leads to a heavy and a light neutrino at the tree level according to the see-saw mechanism, and to one light neutrino mass at the one-loop and the two-loop level, respectively. In the following we will demonstrate that this model is capable of generating a hierarchical mass spectrum fitting well with the mass-squared differences derived from the solar MSW effect and atmospheric neutrino data and that it naturally accommodates large mixing angles corresponding to both mass-squared differences. The fact that tree level and loop neutrino masses appear in our model has an analogy with the models combining the Higgs triplet mechanism with radiative neutrino masses [13,25,26].

## II. THE MODEL

We discuss 3-neutrino oscillations in the framework of an extension of the SM, where a second Higgs doublet ( $\Phi_\alpha$ ,  $\alpha = 1, 2$ ) and a right-handed neutrino singlet  $\nu_R$  are present in addition to the SM multiplets [23]. Thus the Yukawa interaction of leptons and scalar fields is given by

$$-\mathcal{L}_Y = \sum_{\alpha=1}^2 (\bar{L}\Gamma_\alpha\Phi_\alpha\ell_R + \bar{L}\Delta_\alpha\tilde{\Phi}_\alpha\nu_R) + \text{h.c.} \quad (2.1)$$

with  $\tilde{\Phi}_\alpha = i\sigma_2\Phi_\alpha^*$ .  $\Gamma_\alpha$  and  $\Delta_\alpha$  are  $3 \times 3$  and  $3 \times 1$  matrices, respectively. The singlet field  $\nu_R$  permits the construction of an explicit Majorana mass term

$$\mathcal{L}_M = \frac{1}{2} M_R \nu_R^T C^{-1} \nu_R + \text{h.c.}, \quad (2.2)$$

where we assume  $M_R > 0$  without loss of generality.

In this 2-Higgs doublet model, spontaneous symmetry breaking of the SM gauge group is achieved by the VEVs

$$\langle\Phi_\alpha\rangle_0 = \begin{pmatrix} 0 \\ v_\alpha/\sqrt{2} \end{pmatrix}, \quad (2.3)$$

which satisfy the condition

$$v \equiv \sqrt{|v_1|^2 + |v_2|^2} \simeq 246 \text{ GeV}. \quad (2.4)$$

The VEVs  $v_{1,2}$  generate the tree level mass term

$$M_\ell = \frac{1}{\sqrt{2}} \sum_{\alpha=1}^2 v_\alpha \Gamma_\alpha \quad (2.5)$$

for the charged leptons diagonalized by

$$U_L^{\ell\dagger} M_\ell U_R^\ell = \hat{M}_\ell \quad (2.6)$$

with unitary matrices  $U_L^\ell$ ,  $U_R^\ell$  and with a diagonal, positive  $\hat{M}_\ell$ . The most general Majorana neutrino mass term in the model presented here has the form

$$\frac{1}{2} \omega_L^T C^{-1} M_\nu \omega_L + \text{h.c.} \quad \text{with} \quad \omega_L = \begin{pmatrix} \nu_L \\ (\nu_R)^c \end{pmatrix}. \quad (2.7)$$

The left-handed field vector  $\omega_L$  has four entries according to the three active neutrino fields plus the right-handed singlet. The symmetric Majorana mass matrix  $M_\nu$  is diagonalized by

$$U_\nu^T M_\nu U_\nu = \text{diag}(m_1, m_2, m_3, m_4) \quad (2.8)$$

with a unitary matrix  $U_\nu$  and  $m_i \geq 0$ .

### III. THE TREE-LEVEL NEUTRINO MASS MATRIX

From Eqs. (2.1), (2.2) and (2.3) we obtain the tree-level version of  $M_\nu$ :

$$M_\nu^{(0)} = \begin{pmatrix} 0 & M_D^* \\ M_D^\dagger & M_R \end{pmatrix} \quad (3.1)$$

with

$$M_D = \frac{1}{\sqrt{2}} \sum_{\alpha=1}^2 v_{\alpha}^* \Delta_{\alpha}. \quad (3.2)$$

The tree-level mass matrix (3.1) is diagonalized by the unitary matrix

$$U_{\nu}^{(0)} = (u_1, u_2, u_3, u_4) \quad (3.3)$$

with

$$u_{1,2} = \begin{pmatrix} u'_{1,2} \\ 0 \end{pmatrix}, \quad u_3 = i \begin{pmatrix} \cos \vartheta u'_3 \\ -\sin \vartheta \end{pmatrix}, \quad u_4 = \begin{pmatrix} \sin \vartheta u'_3 \\ \cos \vartheta \end{pmatrix}, \quad (3.4)$$

where

$$\tan 2\vartheta = \frac{2m_D}{M_R}, \quad m_D = \|M_D\| = \sqrt{M_D^{\dagger} M_D}. \quad (3.5)$$

The  $u'_{1,2,3}$  form an orthonormal system of complex 3-vectors with the properties

$$u'_{1,2} \perp M_D, \quad u'_3 = M_D/m_D. \quad (3.6)$$

The two non-vanishing mass eigenvalues are given by

$$m_3 = \sqrt{\frac{M_R^2}{4} + m_D^2} - \frac{M_R}{2} \simeq \frac{m_D^2}{M_R} \quad \text{and} \quad m_4 = \sqrt{\frac{M_R^2}{4} + m_D^2} + \frac{M_R}{2} \simeq M_R, \quad (3.7)$$

where the approximate relations refer to the limit  $m_D \ll M_R$ .

#### IV. ONE-LOOP CORRECTIONS AND THE NEUTRINO MASS SPECTRUM

By one-loop corrections, the form of the neutrino mass matrix is changed to

$$M_{\nu}^{(1)} = \begin{pmatrix} \delta M & M_D^* \\ M_D^{\dagger} & M_R \end{pmatrix} \quad (4.1)$$

with  $\delta M$  being a symmetric  $3 \times 3$  matrix. Its explicit form is given by [23]

$$\delta M = \frac{M_R}{8\pi^2} \sum_b \mathcal{M}_b^* \frac{M_b^2 \ln(M_R/M_b)}{M_R^2 - M_b^2} \mathcal{M}_b^{\dagger} + M_D^* \mathcal{A} M_D^{\dagger} \quad (4.2)$$

with

$$\mathcal{M}_b = \frac{1}{\sqrt{2}} \sum_{\alpha=1}^2 b_{\alpha}^* \Delta_{\alpha}. \quad (4.3)$$

In deriving this formula, terms suppressed by a factor of order  $M_D/M_R$  have been neglected. The first term in (4.2) is generated by neutral Higgs exchange. The sum in (4.2) runs over

all physical neutral scalar fields  $\Phi_b^0 = \sqrt{2} \sum_{\alpha=1,2} \text{Re}(b_\alpha^* \Phi_\alpha^0)$ , which are characterized by three two-dimensional complex unit vectors  $b$  [23] with

$$\sum_b b_\alpha b_\beta = \frac{v_\alpha v_\beta}{v^2}. \quad (4.4)$$

Note that we do not consider corrections to  $M_D$  and  $M_R$  in the neutrino mass matrix. The unitary matrix diagonalizing (4.1) can be written in the form [23]

$$U_\nu^{(1)} = U_\nu^{(0)} V \quad (4.5)$$

with  $V - 1$  being of one-loop order. By an appropriate choice of the matrix  $V$  we obtain

$$\hat{M}_\nu^{(1)} = U_\nu^{(1)T} M_\nu^{(1)} U_\nu^{(1)} = \begin{pmatrix} u_1'^T \delta M u_1' & u_1'^T \delta M u_2' & 0 & 0 \\ u_2'^T \delta M u_1' & u_2'^T \delta M u_2' & 0 & 0 \\ 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & m_4 \end{pmatrix}. \quad (4.6)$$

The second term in (4.2) containing the matrix  $\mathcal{A}$  (contributions from  $Z$  exchange and contributions from neutral scalar exchange other than the first term in Eq.(4.2)) cannot contribute to (4.6) because of (3.6). The remaining off-diagonal elements in (4.6) can be removed by choosing  $u_1'$  orthogonal to  $\Delta_1$  and  $\Delta_2$ . This shows at the same time that one of the neutrinos remains still massless at the one-loop level. However, there is no symmetry enforcing  $m_1 = 0$  and the lightest neutrino will in general get a mass at the two-loop level [27]. Finally, the vector  $u_2'$  has to be orthogonal to  $u_1'$  and  $u_3'$ , and its phase is fixed by the positivity of  $m_2$ . Defining

$$c_\alpha = \frac{v}{\sqrt{2} m_D} u_2'^\dagger \Delta_\alpha, \quad (4.7)$$

the relation  $m_D = \|M_D\|$  implies  $v_1^* c_1 + v_2^* c_2 = 0$ , but the quantity  $|c_1|^2 + |c_2|^2$  remains an independent parameter of our model, only restricted by “naturalness”, which requires that it is of order 1. From (4.2), using the Cauchy–Schwarz inequality and  $\|b\| = 1$ , we obtain the upper bound

$$m_2 \leq \frac{m_D^2}{8\pi^2 M_R} \sum_b \frac{\ln(M_R/M_b)}{1 - M_b^2/M_R^2} \frac{M_b^2}{v^2} (|c_1|^2 + |c_2|^2). \quad (4.8)$$

Note that cancellations in Eq.(4.2) in the summation over the physical neutral scalars do not happen in general because the vectors  $b$  are connected with the diagonalizing matrix of the mass matrix of the neutral scalars. The elements of these matrix are independent of the masses  $M_b^2$  (see, e.g., Ref. [28]). From these considerations it follows that the order of magnitude of  $m_2$  can be estimated by

$$m_2 \sim \frac{1}{8\pi^2} m_3 \frac{M_0^2}{v^2} \ln \frac{M_R}{M_0}, \quad (4.9)$$

where  $M_0$  is a generic physical neutral scalar mass. Note that for  $M_0 \sim v$  the relation  $m_2 \ll m_3$  comes solely from the numerical factor  $1/8\pi^2$  appearing in the loop integration.

## V. DISCUSSION

Let us first discuss the neutrino mass spectrum in the light of atmospheric and solar neutrino oscillations. Due to the hierarchical mass spectrum in our model we have

$$\Delta m_{\text{atm}}^2 \simeq m_3^2 \quad \text{and} \quad \Delta m_{\text{solar}}^2 \simeq m_2^2. \quad (5.1)$$

From the atmospheric neutrino data, using the best fit value of  $\Delta m_{\text{atm}}^2$ , one gets [3]

$$m_3 = \frac{m_D^2}{M_R} \simeq 0.06 \text{ eV}. \quad (5.2)$$

A glance at Eq.(4.9) shows that  $m_2$  is only one or two orders of magnitude smaller than  $m_3$  if  $M_R$  represents a scale larger than the electroweak scale. Therefore, our model cannot describe the vacuum oscillation solution of the solar neutrino problem. On the other hand, with the MSW solution one has [4]

$$m_2 \sim 10^{-2.5} \text{ eV} \quad (5.3)$$

and, therefore,  $m_2/m_3 \sim 0.05$ , which can easily be achieved with Eq.(4.9). In principle, the unknown mass scales  $m_D$  and  $M_R$  are fixed by Eqs.(5.2) and (5.3) (see Eq.(4.9) for the analytic expression of  $m_2$ ). However, due to the logarithmic dependence of Eq.(4.9) on  $M_R$  and the freedom of varying the scalar masses, whose natural order of magnitude is given by the electroweak scale, the heavy Majorana mass could be anywhere between the TeV scale and the Planck mass.

Let us therefore give a reasonable example. Assuming that  $m_D$  has something to do with the mass of the tau lepton, we fix it at  $m_D = 2 \text{ GeV}$ . Consequently, from Eq.(5.2) we obtain  $M_R \simeq 0.7 \times 10^{11} \text{ GeV}$ . Inserting this value into Eq.(4.9) and using (5.3), the reasonable estimate  $M_0 \sim 100 \div 200 \text{ GeV}$  ensues, which is consistent with the magnitude of the VEVs. This demonstrates that our model can naturally reproduce the mass-squared differences needed to fit the atmospheric and solar neutrino data, where the fit for the latter is done by the MSW effect.

Now we come to the mixing matrix (1.1), which is given by

$$U = U_L^{\ell\dagger} U'_\nu \quad \text{with} \quad U'_\nu = (u'_1, u'_2, u'_3) \quad (5.4)$$

for  $M_R \gg m_D$  (see Eq.(3.4)) and neglecting  $V$  (4.5). Since the directions of the vectors  $\Delta_{1,2}$  in the 3-dimensional complex vector space determine  $U'_\nu$ , we will have large mixing angles in this unitary matrix as long as we do not invoke any fine-tuning of the elements of  $\Delta_{1,2}$ . (This is in contrast to Ref. [23] where we assumed that  $(U_L^{\ell\dagger} \Delta_\alpha)_j \sim m_{\ell j}/v$ , where the  $m_{\ell j}$  are the charged lepton masses.) Also  $U_L^\ell$  might have large mixing angles, but could also be close to the unit matrix in analogy to the CKM matrix in the quark sector. Since we do not expect any correlations between  $U_L^\ell$  and  $U'_\nu$ , it is obvious that our model favours large mixing angles in the neutrino mixing matrix  $U$ .

On the other hand, there is a restriction on the element  $U_{e3}$  from the results of the Super-Kamiokande atmospheric neutrino experiment and the CHOOZ result [29] (absence of  $\nu_e^{(-)}$

disappearance), which is approximately given by [30]  $|U_{e3}|^2 \lesssim 0.1$ . Furthermore, the Super-Kamiokande results imply that  $\sin^2 2\theta_{\text{solar}}$  is close to 1 (see introduction). These restrictions find no explanation in our model, but, as we want to argue, not much tuning of the elements  $U_{a3} = (U_L^\dagger u'_3)_a$  is needed to satisfy them. If we take the ratios  $|U_{e3}| : |U_{\mu 3}| : |U_{\tau 3}| = 1 : 2 : 2$  as an example we find  $|U_{e3}|^2 = 1/9 \simeq 0.11$  and Eq.(1.3) gives  $\sin^2 2\theta_{\text{atm}} = 80/81 \simeq 0.99$ . To show that the favourable outcome for  $\sin^2 2\theta_{\text{atm}}$  does not depend on having  $|U_{\mu 3}| \simeq |U_{\tau 3}|$ , let us consider now 1:3:2 for the elements  $|U_{a3}|$ . Then we obtain  $|U_{e3}|^2 = 1/14 \simeq 0.07$  and  $\sin^2 2\theta_{\text{atm}} = 45/49 \simeq 0.92$ . Thus not much fine-tuning is necessary to meet the restrictions on  $|U_{e3}|^2$  and  $\sin^2 2\theta_{\text{atm}}$  [10,15,31]. Obviously, our model would be in trouble if it turned out that the atmospheric and solar neutrino oscillations decouple with high accuracy ( $U_{e3} \rightarrow 0$ ) or atmospheric mixing is *very* close to maximal.

Since there is no lepton number conservation in the present model, lepton flavour changing processes are allowed. The branching ratios of the decays  $\mu^\pm \rightarrow e^\pm \gamma$  and  $\mu^\pm \rightarrow e^\pm e^+ e^-$  have the most stringent bounds [32]. With our assumption on the size of the Yukawa couplings  $\Delta_\alpha$ , the contribution of the charged Higgs loop [2] to  $\mu^\pm \rightarrow e^\pm \gamma$  leads to a lower bound of about  $100m_D$  for the charged Higgs mass. The decay  $\mu^\pm \rightarrow e^\pm e^+ e^-$ , proceeding through neutral Higgs scalars at the tree level, restricts only some of the elements of the Yukawa coupling matrices  $\Gamma_\alpha$ , but not those of the  $\Delta_\alpha$  couplings relevant in the neutrino sector. It is well known that the effective Majorana mass relevant in  $(\beta\beta)_{0\nu}$  decay is suppressed to a level below  $10^{-2}$  eV in the 3-neutrino mass hierarchy [33], which is considerably smaller than the best present upper bound of 0.2 eV [34].

In summary, we have discussed an extension of the Standard Model with a second Higgs doublet and a neutrino singlet with a Majorana mass being several orders of magnitude larger than the electroweak scale. We have shown that this model yields a hierarchical mass spectrum  $m_3 \gg m_2 \gg m_1$  of the three light neutrinos by combining the virtues of seesaw ( $m_3$ ) and radiative neutrino mass generation ( $m_2 \neq 0$  and  $m_1 = 0$  at the one-loop level), and that it is able to *accommodate* easily the large mixing angle MSW solution of the solar neutrino problem and the  $\nu_\mu \rightarrow \nu_\tau$  solution of the atmospheric neutrino anomaly. By construction, the neutrino sector of our model is very different from the charged lepton sector. The model offers no explanation for the mass spectrum of the charged leptons. We want to stress that the scalar sector of the model is exceedingly simple and that – apart from the Standard Model gauge group – no symmetry is involved. The moderate smallness of  $|U_{e3}|^2$  and closeness of  $\sin^2 2\theta_{\text{solar}}$  to 1 is controlled by the ratios of the elements of the third column of the mixing matrix  $U$ . We have argued that the ratios of  $|U_{a3}|$  ( $a = e, \mu, \tau$ ) required to give  $|U_{e3}|^2 \lesssim 0.1$  and  $\sin^2 2\theta_{\text{solar}} \gtrsim 0.84$  are quite moderate, with 1:2:2 being a good example. Such suitable ratios have to be assumed in the model presented here, but might eventually find an explanation by embedding it in a larger theory relevant at the scale  $M_R$ .

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